Virtual Team Challenge: **Spill!**
Supplemental Curriculum
Problem Solving Handout #1

Problem Solving in Business: Simple Modeling

What is a Model? A 'model' is a representation of reality. Even though the word may seem fancy, chances are, you've used them all your life. Perhaps you have developed models of your classmates' behavior without even knowing it. Perhaps you subconsciously use this model, and can anticipate which of them would be likely help with a homework problem, or which would be likely to lend you $10 for lunch. The act of developing models is known as 'modeling'.

Many different types of businesses use different types of models to represent reality. Architectural firms use *physical* models to help decision-makers determine how a building may look. Engineers sometimes build *small scale models* to make sure that a new plant will operate as designed. Software developers use *schematic models* or pictures, drawings, and charts to describe the functionality of their products.

Businesses typically use *mathematical models* that represent reality as they make decisions and business plans. If you have ever seen a formal, written business plan, it is likely that it contained some mathematical models. Here, we consider some simple mathematical models, and give you tools to answer questions based on mathematical models.

A Simple Example: The Profit and Loss Model
Do you remember the definition of profit and loss? Profits are the amount by which revenues exceed expenses, and losses are the amounts by which expenses exceed revenue. This is how we would describe this relationship within a model:

$$\text{Revenues} - \text{Expenses} = \text{Profit (or Loss, if negative)}$$

Using this mathematical model as a basis, we can use the laws of arithmetic to deduce other relationships:

$$\text{Revenues} = \text{Profit (or Loss, if negative)} + \text{Expenses}$$

$$\text{Expenses} = \text{Revenues} - \text{Profit (or Loss)}$$

In business, mathematical modeling is almost always a series of equations or inequalities used as tools for making decisions. We can use laws of arithmetic and algebra to develop models, and to solve them.

Steps in Solving Problems Based on Mathematical Models
In solving problems based on mathematical models, it is helpful to take the following steps:

1) *Develop a Model.* Choose or invent a model that describes the realities you are facing. If you need to determine what profit a business will have this year, perhaps the profit model would be appropriate.
2) *Determine the Data for the Model.* Determine what information your model should consider. If you are trying to determine profit, you need to determine fully determine the appropriate information: revenues and expenses for example.
3) *Solve the Model.* Using the data you have input, solve the model for the wanted variable. In determining the profit or loss of a business, we would solve for the profit variable.
4) *Check your Results.* Work through the model again and determine that your solution is appropriate. Once you have checked your answer, your model is complete.

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Problem Solving Handout #2

Problem Solving in Business: More Complex Modeling

More Complex Models. In the first module, we considered *mathematical modeling*, or the act of developing representations of reality based on equations and inequalities. Now, we shall take a look at the model to which you have been introduced, and provide you with other mathematical models that express revenues and expenses – and in turn, profit and loss.

The Profit/Loss Model and its Components. Profits are the amount by which revenues exceed expenses, and losses are the amounts by which expenses exceed revenue. This is how we described this relationship:

$$\text{Revenues} - \text{Expenses} = \text{Profit (or Loss, if negative)}$$

Within this model, we have two components – revenues and expenses – which can also be described as mathematical models.

Revenue. Revenue refers to the income a business receives from its usual, ongoing business activities. Most often, revenue is derived from selling goods and/or services to customers. In many cases, we can express this as the price per unit of goods or services, times units sold:

$$\text{Revenues} = \text{Price per unit} \times \text{Number of Units Sold}$$

For example, if Acme Widgets sells 800,000 units at $5 each, its revenues are $4,000,000.

Expenses. Expenses refers to the resources a business must use in order to produce revenue. In many cases, this is expressed as the summation of fixed costs, and variable costs:

$$\text{Expenses} = \text{Fixed Costs} + \text{Variable Costs}$$

Similarly, variable costs are sometimes expressed as the cost it takes to acquire or produce each unit sold, times the total units sold:

$$\text{Variable Costs} = \text{Cost Per Unit} \times \text{Units Sold}$$

And so we can also express expenses as follows:

$$\text{Expenses} = \text{Fixed Costs} + (\text{Cost Per Unit} \times \text{Units Sold})$$

So for example, if Acme Widgets must spend $1,500,000 in fixed costs, and must spend $1 on each of the 2,000,000 units it sells, then its expenses are $3,500,000.

Putting it Together. Now, using the laws of arithmetic, we have a complete model to describe the profits, losses, revenues, and expenses for Acme Widgets:

$$\text{Profit or Loss} = (\text{Price per unit} \times \text{Number of Units Sold}) - (\text{Fixed Costs} + (\text{Cost Per Unit} \times \text{Units Sold}))$$

Example. *Acme Widgets has recently expanded and now has $2,500,000 in fixed costs. It must spend $1.50 to produce each of the 1,000,000 widgets it will sell this year. The average selling price of its widgets is $5. Describe the profits, losses, revenues, and expenses for Acme Widgets. What are it total profits?*

$$\text{Profit or Loss} = (5 \times 1{,}000{,}000) - (2{,}500{,}000 + (1.50 \times 1{,}000{,}000)) = 5{,}000{,}000 - 4{,}000{,}000 = 1{,}000{,}000$$

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Supplemental Curriculum
Problem Solving Handout #3

Concepts in Cost-Benefit Analysis: Return on Investment

The Investment Decision. Businesses make decisions in order to achieve their goals. One such decisions that businesses make is the 'investment decision', which is a decision about where to invest resources to achieve the highest tangible gains, or in other words, highest 'return'. Decision-makers use a ótreturn on investmentô analysis to evaluate such potential investments and help them choose between them.

Consider the example of a widget factory that earns money each year by profitably selling widgets to customers. A typical investment decision would be whether to expand operations to produce more widgets, or invest the earnings it has retained over the years into other investments such as new plants, investment property, or entirely new business ventures in either a new or related industry.

Return on Investment (ROI) Analysis is a tool that businesses use while making investment decisions. It is a variation of cost-benefit analysis, where a potential investment is evaluated by comparing the costs of the investment with the gains expected to result from it. To perform this evaluation, they first determine the full cost of the investment, and then project the benefits or returns that will accrue from of the investment.

Acme Widgets is facing an investment decision. It must choose whether to expand operations and produce more widgets, or invest in an entirely new business venture. Decision-makers at Acme may want to calculate returns for both scenarios to help them choose.

Calculating Return on Investment. Return on investment is a ratio. It is meant to measure the returns, or benefits, provided by an initial investment in relation to its initial investment costs. It is calculated by comparing the gains from an investment as follows:

$$\text{Return on Investment (ROI)} = \frac{\text{Gains from Investment} - \text{Investment Costs}}{\text{Investment Costs}}$$

This ratio describes what proportion of the initial value or cost of the investment is 'returned' by proceeds that accrue from the investment.

Example. *Acme Widgets is deciding whether or not to pursue a new business venture. It would likely yield $1MM in profits over each of the next twelve months, and cost $7MM to startup and $3MM to operate for one year. What is the* ***return on investment*** *from this venture in the first year?*

$$\text{ROI} = \frac{12{,}000{,}000 - 10{,}000{,}000}{10{,}000{,}000} = \frac{2{,}000{,}000}{10{,}000{,}000} = \frac{1}{5} \text{ or } 20\%$$